Mathematics and Politics
Prepared by Dr.Sudeshna Basu and Timothy B.Clarke
Two days per week, 75-minute class meetings

Possible models for team teaching (switch off depending on unit, switch off T/Th, switch off T/Th while in different units, consider recording lectures)

Textbook:
The Mathematics of Politics – E. Arthur Robinson and Daniel Ullman
Publisher: CRC Press

Assignment Types – give details and descriptions about when/where the D-J components thread in
In-class activities
Homework
Quizzes
Exams (after Voting Systems, Apportionment, Final Exam)

Timeline (26 lectures, 2 exams)
Part I: Voting Systems (9 classes, 4.5 weeks)
        Most non-trivial systems, criteria, comparing voting systems, Arrow's Theorem
        Chapters: 1, 2 (Keep three methods, Plurality, Borda, Copeland?), 3, 4 (slimmed due to Chapter 2 choices), 5

Part II: Apportionment (9 classes, 4.5 weeks)
        Most non-trivial systems, criteria, comparing, Impossibility Theorem
        Chapters: 7, 8 (Keep four methods, Jefferson, Adams, Hill, Webster?), 9, 12 (students read and write about this)

Part III: Redistricting (4 classes, 2 weeks)
        Supplemental materials on district creation and gerrymandering

Part IV: Electoral College (4 classes, 2 weeks)
        Chapters: 19, 20

Week	Mathematics	Democracy	Diversity-Justice	Notes
Election to a single seat or office, decision on an issue (Chapters 1, 2, 3, 4, 5)	Distinguish between majority, plurality, unanimous, weighted, supermajority, tie breaking, recall.	What constitutes fairness in elections? Which voters and candidates benefit or are harmed in voting systems?	**Focusing Question:** How do different voting systems support voter rights or disenfranchise voters? Disenfranchise: to take away power or opportunities, especially the right to vote, from a person or group
Week 1	Voting Systems: Two Candidates/Choices (1)	Direct	Why is one voting system "fair" than the other from the perspectives of the voter or the perspectives of the candidates. How to strike a balance between the voters and candidates. For example, the simple majority is nearly the best voting method since it treats all voters equally and all candidates equally, as opposed to weighted voting favoring certain voters over the other or the Status quo method favors one candidate over the other.	Start with scenarios/familiar contexts, draw examples from Student organization election in the university.
2	Voting Systems: Social Choice Functions (2)	Individual elections	A description of the process of choosing the winner for an election with more than two candidates. To bring in	Start with scenarios/familiar contexts, draw examples from Student organization election in the university, look at contemporary elections.

			fairness the different methods look at ranking of the voters as opposed to only their first choice. Overall, this brings in more fairness from the perspectives of both the voters and the candidates.	
3	Voting Systems: Criteria for Social Choice (2)		How fair is a voting system in general from a voter's perspective and a candidate's' perspective? How fair is a system in deciding a winner based on the voter's choices? How fair is a voting system to the voters, to the candidates? How fairly is the ranking of the voter's choice reflected in deciding the winner?	Look at actual scenarios and different voting systems and Examine what criteria are satisfied.
4	Voting Systems: Which Methods are Good? (2)	Compromise, no perfect method	Making the ultimate decision on which methods are most fair for both the voters and candidates. What is the trade off?	In-class activity: Instant run-off and/or ranked choice
5	Voting Systems: Arrow's Theorem (1) Exam #1 (1)			Exam #1
How many seats in a representative body/how many districts in a municipality (7, 8, 9, 12 – project)	Distinguish between standard divisor methods	Which states benefit or are harmed in apportionment systems?	**Focusing Question:** Which states do different methods favor? Population

				and modified divisor methods.		Density comparison: Delaware, Wyoming, and Montana.
Week 6	Apportionment: Hamilton's Method (1.5)	Representative	How fair is this apportionment method? What happens when the population of a state increases or decreases, does this method do justice to the state? If a new state is added to the system does the method behave fairly to the older states? If the number of seats is increased do each state retain their original apportionment?	Standard Divisors		
7	Apportionment: Hills and other Divisor Methods (2)	Number of seats proportional to population (US House)	How fair are these methods from different perspectives? Do these methods satisfy upper and lower quota rules? What happens when the number of seats increases in the house, does each state retain its original apportionment? What happens if the population of a state increases or decreases? Do these methods take care of the apportionment accordingly?	Modified Divisors		
8	Apportionment: Criteria and Impossibility (2.5)		Are these methods fair from the perspectives of			

			states? What happens if two states exchange their population, is their apportionment is interchanged? If a state has more population than another state does the bigger state have more apportioned seats than the smaller state? Are these methods fair to different censuses where the population distribution is the same?	
9	Apportionment: History, Growing with Population Growth (2)		Is 435 house members appropriate in 2024? Fair? Compare historical membership counts and state/national populations.	
10	Exam #2 (1)			How large should the House of Representatives be?
How do districts represent voters? (supplemental materials)		Examine and compare alternative maps in a selected number of states.	Which voters and elected officials benefit or are harmed in districting systems?	**Focusing Question:** Should voters choose candidates? Should candidates choose voters?
11	Redistricting: Concept, Criteria, Geometry	Representative districts are created based on population and geography.	Which geographic aspects of a community should we use when drawing districts? How do redistricting choices affect voters and candidates?	Lots of external reading here.

12	Redistricting: Gerrymandering, Cracking and Packing	Geographic partition based on the number of seats	How can the techniques of packing and cracking be used to dilute voter power and consolidate candidate (or party) power?	
How does the Electoral College work? (Chapters 19, 20)		Compare state election results and electoral college votes. Distinguish between winner-take all allocation and alternate allocation methods. (Maine, Nebraska, popular vote split)	Which voters benefit or are harmed by the Electoral College?	**Focusing Question:** How do different electoral college methods support voter rights or disenfranchise voters across states?
13	Electoral College: Weighted Voting (2)	How can Electoral College results amplify popular vote counts? How can they subvert popular vote counts?	What historical contexts inform the creation of the Electoral College? What historical aspects of the Electoral College persist?	
14	Whose advantage? (2)	Which voters and states are underrepresented in Electoral College Counts? Which are overrepresented?	How do population disparities positively or negatively affect a state's impact on the Electoral College? To whom is the Electoral College fair and unfair?	
15	Final Exam			

Textbook Homework Assignments

Assignments will give students practice with the quantitative aspects of electoral systems and prompt them to attend to diversity and justice issues by addressing fairness of the electoral systems using various criteria.

Chapter 1	1.10,1.12
Chapter 2	2.6,2.8
Chapter 3	3.2,3.18
Chapter 4	4.2,4.5,4.14
Chapter 5	5.5,5.8
Chapter 7	7.2,7.4,7.8
Chapter 8	8.6,8.8,8.10
Chapter 9	9.5,9.11
Chapter 11	11.1,11.2,11.4
Chapter 19	19.8, 19.10
Chapter 20	206 ,20.7

In-Class Activities (similar to textbook problems, but conducted as group activities)

Voting Systems: Students will determine the outcome of elections according to different voting systems.

Apportionment: Students will determine the results of apportionment according to different methods.

Redistricting: Students will demonstrate the most common methods of redistricting and gerrymandering. Students will explore the district maps of one of the United States and analyze it for evidence of gerrymandering.

Electoral College: Students will determine the outcome of different elections according to the rules of the Electoral College.

Final Paper

Students will write a 3–5 page paper in which they:

- Identify the electoral layers of democracy in the United States and describe how systems at each layer enfranchise or disenfranchise voters.
- Identify historical aspects of the electoral layers of democracy in the United States and recognize the diversity of past and ongoing struggles for representation.
- Analyze the electoral components of democracy in the United States with an eye toward structural sources of injustice.

Learning Aims

Part I: Voting Systems

- Describe the assumptions, components, and characteristics necessary for creating voting systems.
- Compare and contrast the results of different voting systems given voter data about candidate preferences.
- Examine criterion for fairness in voting systems with an eye towards strengths and weaknesses of individual systems.
- Identify impossibilities, paradoxes, and compromises that result from different voting systems.
- Decide whether and how a given social choice function meets a given voting criterion.

Part II: Apportionment

- Describe the assumptions, components, and characteristics necessary for creating apportionment systems.
- Compare and contrast the results of different apportionment systems.
- Examine criterion for fairness in apportionment systems with an eye towards strengths and weaknesses.
- Decide whether and why a given apportionment method meets a given apportionment criterion.

Part III: Redistricting

- Describe the assumptions, components, and characteristics necessary for creating electoral districts for use in voting systems.
- Compare and contrast the results of different methods for the creation of electoral districts.
- Examine criterion for fairness in districting methods with an eye towards strengths and weaknesses.
- Decide whether and why a given districting method meets a given districting criterion.
- Describe how Gerrymandering can be used to influence election results.

Part IV: Electoral College

- Summarize the features of the Electoral College.
- Analyze the advantages and disadvantages of Electoral College and realize the importance of voters in the process.

Domestic Diversity + Justice Aims

- Identify the electoral layers of democracy in the United States and describe how systems at each layer enfranchise or disenfranchise voters.
- Identify historical aspects of the electoral layers of democracy in the United States and recognize the diversity of past and ongoing struggles for representation.
- Analyze the electoral components of democracy in the United States with an eye toward structural sources of injustice.

D-J Themes

- Voting types: Referendum by popular vote, Jury voting, State voting systems, Electoral college (small state, big state, swing state).
- Laws governing voter registration, ballot access and restrictions, recounts of election results.
- Apportionment: 3/5 compromise, under-representation (U.S. House of Representatives – states with large populations who have high voter to representative ratios, Washington D.C. – no voting representative in House or Senate), and over-representation (States with small populations have same number of senators as states with large populations).
- Gerrymandering: hyper-partisan outcomes, disproportionate results for minority groups.

University Diversity and Justice Aims

Domestic:

Students will demonstrate a meaningful understanding of structural sources of disenfranchisement and privilege in a domestic context.

Justice:

Students will demonstrate a meaningful understanding of the history, perspectives, values, and methods of at least one justice- oriented movement or intellectual tradition.

Students will demonstrate a meaningful understanding of structural sources of injustice.

> **Commented [AK1]:** I like that you have this as a bullet-point in your learning aims: I would recommend writing these out similarly to the learning aims you listed above. I also recommend categorizing these specific learning aims under Domestic and/or Justce for further clarity.

Mode of operating: Aim for fairness, describe criteria for fairness, investigate how criteria can coexist, encounter impossibilities, settle for compromise…

At the outset of each unit, start with a discussion of student knowledge and scenarios that serve as sites for future mathematical discussion.

1.0 Scenario: put different systems/views on cards and then ask students to consider them in groups…

Contexts: elections, juries, ownership/equity in companies

Psychology, jury composition, etc.

Possible assignments with examples that show the product of student thinking and instructor emphasis on D-J
- History reading and writing (from textbook)
- Article reflections from current events (gerrymandering, etc.)
- Projects that engage students about their own representatives and regions

Resources:

https://en.wikipedia.org/wiki/National_Popular_Vote_Interstate_Compact

https://www.smithsonianmag.com/smart-news/why-do-maine-and-nebraska-split-their-electoral-votes-180976219/

https://www.census.gov/newsroom/blogs/random-samplings/2021/04/how-apportionment-is-calculated.html

https://apnews.com/article/supreme-court-redistricting-race-voting-rights-alabama-af0d789ec7498625d344c0a4327367fe

https://apportionment.app/

https://www.youtube.com/watch?v=HoAnYQZrNrQ

https://en.wikipedia.org/wiki/United_States_congressional_apportionment

https://projects.fivethirtyeight.com/redistricting-maps/

Commented [TC2]: @Timothy Clark Organize by section

https://www.technologyreview.com/2021/08/12/1031567/mathematicians-algorithms-stop-gerrymandering/

https://www.theguardian.com/us-news/2021/aug/22/gerrymandering-us-electoral-districts-congress

https://usafacts.org/visualizations/electoral-college-states-representation/

https://www.newsweek.com/states-where-individual-voters-have-most-impact-electoral-college-1545214
https://www.pewresearch.org/short-reads/2023/09/25/majority-of-americans-continue-to-favor-moving-away-from-electoral-college/